3. EXERCISE SHEET, RETURN DATE MAY 21ST 2015

General notes

50% of the exercises, that involve no programming have to be treated. Handing in is in groups now.

Exercise 1 (1 point)

Derive Heun's method by numerical integration of

$$y'(t) = f(t, y(t)),\ y(0) = y_0$$

on a uniform grid of size h using the trapezoidal rule.

Which numerical integration scheme corresponds to the explicit Euler method and the modified Euler method?

Exercise 2 (2 points)

Which conditions on the entries of the general Butcher's tableau

$$\begin{array}{c|cc} 0 & & \\ c_2 & a_{21} & \\ \hline & b_1 & b_2 \end{array}$$

should hold, such that the corresponding explicit Runge-Kutta method is consistent of order two?

Exercise 3

Consider the following second order differential equations, which describe a mass on a spring and the mathematical pendulum respectively:

$$\ddot{x}(t) = -x(t),\ \ x(0) = 1,\ \ \dot{x}(0) = 0$$

$$\ddot{\alpha}(t) = -sin(\alpha(t)),\ \ x(0) = 1,\ \ \dot{\alpha}(0) = 0$$

Here the second derivatives correspond to the accelleration. By Newton's laws of motion the accelleration is proportional to the force acting on the point mass. In the first equation x is the displacement of the mass from the equilibrium position. α is the angle of the pendulum. Call $\nu = \dot{\alpha}$ the angular velocity of the pendulum and $v = \dot{x}$ the velocity of the mass on the spring.

Reformulate the equations above to obtain a system of first order differential equations.

Solve this system using your program with the three methods from the previous homework sheet and plot the solutions. What happens to the solutions, if the step size increases?

Note: the first differential equation above allows you to evaluate trigonometric functions numerically. What other methods do you know to do so?

What would you expect, if you solved the pendulum equation with $\alpha(0) = \pi = -\pi,\ \ \dot{\alpha}(0) = 1$. Solve it and plot it.

Exercise 4 (optional)

Visualize the pendulum and the mass point on the spring.