# 4. EXERCISE SHEET, RETURN DATE MAY 28TH/29TH

(1st revision)

## General notes

In the exercise we used $p$ for position. This was a rather bad choice because this letter is reserved for the momentum $p = m \cdot v$. We will use $q$ for position from now on.

With

```
glPointSize(2.0f);
        glBegin(GL_POINTS);
                glVertex3d(..., ..., 0.0);
        glEnd();
```

you can draw points (actually little squares) that are 2 pixels in size.

The example inputHandling.c on the website shows how to handle keyboard and mouse input to interact with the graphics. It also uses the call

```
glMatrixMode(GL_PROJECTION);
        glLoadIdentity();
        glFrustum(-1.0, 1.0, -1.0, 1.0, 1.5, 10000.0);
```

to set up a projective transformation. With this only objects with $z$-coordinate

$$-10000 < z < -1.5$$

are visible. Thus you have to move everything by calling e.g.

```
glTranslated(0.0, 0.0, -5.0);
```

The example also shows how to redraw the image in regular intervals of 10 milliseconds.

## Exercise 1

Derive a method for step size control, for example the 'Einbettungsverfahren' described in section 1.7.1 of the Skript or one of your choice. Test it on $y'(x) = -y(x) \cdot x$ and $y'(x) = 1/(1-x)$.

## Exercise 2

Write a method to draw the numerically calculated orbit of a second order ODE in a position-velocity diagram. Test this on the pendulum and mass-on-a-string system.

Use Euler, symplectic Euler, Heun, Runge-Kutta of 4th order and the adaptive scheme from exercise 1 above. Thoroughly play with different initial values and step sizes.

In particular test what happens if the pendulum is started with an angle of nearly $\pi$ and zero velocity. In this case, is an adaptive step size appropriate to account for the high curvature in the position-velocity diagram?

## Exercise 3

For simulating a spacecraft in the gravitational field of the earth, solve the differential equation

$$\dot{p}(t) = v(t)$$
$$\dot{v}(t) = -\frac{p}{||p||^3} + F$$

where $p$ and $v$ are both two dimensional. $F$ shall be $\begin{pmatrix} 0 \\ 0.1 \end{pmatrix}$ if the W-key is pressed, $\begin{pmatrix} 0 \\ -0.1 \end{pmatrix}$ if the S-key is pressed and $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ otherwise. Draw only the current position of the spacecraft on the screen and a little point at the origin. Adjust

```
SDL_Delay(int i);
```

appropriately so that the variable $t$ corresponds to the real time.

Is a variable step size appropriate for this problem? If so, why?

## Exercise 4 (1 point)

When solving the mass-on-a-spring system using the symplectic Euler method you can observe that the deviation of the numerically calculated orbit from the exact one stays bounded independent of the number of steps. Try step sizes 1, 1.3, 0.8, 0.5 and 0.1 and watch the orbit (The lines connecting the points in the position-velocity diagram do not belong to the orbit, so plot points instead of lines for this purpose). You can easily calculate a Hamiltonian that stays constant under the symplectic Euler step and thus you can find the exact deviation of the orbit from the orbit of the exact Hamiltonian $H = q^2 + v^2$.

## Exercise 5 (optional)

Implement an implicit scheme for the problem in exercise 3.