5. EXERCISE SHEET, RETURN DATE JUNE 11TH/12TH

Exercise 1

Solve the following initial value problem using the explicit Euler method and plot the solution up to $t = 10$:

$$x'(t) = -2 \cdot x$$

$$y'(t) = -2 \cdot y$$

$$x(1) = 3 \cdot \mathrm{e}^{-2} + 0.001$$

$$y(1) = 3 \cdot \mathrm{e}^{-2}$$

Use stepsizes $h = 0.1$, $h = 0.8$, $h = 1$ and $h = 2$.

Exercise 2

Solve the following initial value problem using the explicit Euler method and plot the solution up to $t = 10$:

$$x'(t) = -1001 \cdot x(t) + 999 \cdot y(t)$$

$$y'(t) = 999 \cdot x(t) - 1001 \cdot y(t)$$

$$x(1) = 3 \cdot \mathrm{e}^{-2} + 0.001$$

$$y(1) = 3 \cdot \mathrm{e}^{-2}$$

Use stepsizes $h = 0.1$, $h = 0.8$, $h = 1$ and $h = 2$.
Ok, try again with step sizes $h = 0.0001$, $h = 0.000999$, $h = 0.001$, $h = 0.0010005$, $h = 0.002$.
Also try $h = 0.00102$ and only plot from $t = 1$ to $t = 1.2$ in a large window.

Exercise 3

Compare the exact solutions of the initial value problems from exercise 1 and 2. Calculate the stiffness quotients of the differential equations from exercise 1 and 2.

Exercise 4

Try to solve the Lorenz differential equations numerically. I suggest using a variable step size method of high order. Convince yourself of the correctness of the solution by showing that the number of windings around each of the two parts of the attractor stays constant under a decrease in step size tolerance. The differential equation is the following:

$$x'(t) = \sigma \cdot (y(t) - x(t))$$
$$y'(t) = x(t) * (\rho - z(t)) - y(t)$$
$$z'(t) = x(t) * y(t) - \beta \cdot z(t)$$

where $\sigma = 10$, $\rho = 28$ and $\beta = \frac{8}{3}$. The initial value is $x(0) = 0.1$, $y(0) = 0$, $z(0) = 0$. For the 3D-plot of the solution you can use the example code lorenz.c from the website.

Exercise 5 (optional)

Implement an implicit scheme for the problem in exercise 3 of the 4th homework sheet.

Exercise 6 (optional)

Extend your spaceship program to the situation in which the spaceship has the mass of the earth.