7. EXERCISE SHEET, RETURN DATE JULY 2./3.

Important Note

Don't worry, there will be sheet 8 and sheet 9 soon.

Exercise 1 (optional)

In this exercise we want to investigate the behaviour of Euler's equations, which come up in the dynamics of rigid bodies. The equation describes the motion of the axis of rotation in 'Körperkoordinaten'. Call this axis $\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$. Then the differential equations read

$$\dot{\omega_1} = \omega_2 \omega_3 \cdot \frac{J_2 - J_3}{J_1}$$
$$\dot{\omega_2} = \omega_1 \omega_3 \cdot \frac{J_3 - J_1}{J_2}$$
$$\dot{\omega_3} = \omega_1 \omega_2 \cdot \frac{J_1 - J_2}{J_3}.$$

Which numerical method suits this differential equation best? Do we need step-size control? Is there a way to conserve energy? When you have found the function that stays constant under the flow of this ODE (Hint: ellipsoid, maybe there are others), how can you ensure, that your numerical method conserves this energy as well?

Plot the solution curves in three dimensions.

Find some way of changing $J = \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix}$ and ω in real time, without having to restart the program (Suggestion: Mouse axes for Euler angles, mouse key to switch between J and ω, keyboard keys for length of the vectors).

Exercise 2 (optional)

The motion of a rigid body is described by the following ODE

$$\dot{Q} = Q \cdot W$$

where Q is the orthonormal rotation matrix of positive determinant of the rigid body around its centre of mass and $W = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}$ its derivative in 'Körperkoordinaten'. Note that Q stays orthonormal under the effect of this ODE. How can you guarantee this for the numerical solution? (Hint: same method as in exercise 2, sheet 6, maybe need Newton method for this, maybe enough to use one Newton step)

Exercise 4 (optional)

Implement an implicit scheme for the problem in exercise 3 of the 4th homework sheet.

Exercise 5 (optional)

Use Taylor expansion to derive finite difference quotients for the first derivative with an order of consistency of 1, 2 and 3.

Exercise 6 (optional)

Extend your spaceship program to the situation in which the spaceship has the mass of the earth, or:

Extend your spaceship program to the situation in which there is a moon. Can you make the space ship orbit in a figure 8 around the two bodies. Remember that step size control is essential for attaining accurate results. As this three-body problem produces an ODE in 12 dimensions. Consider implementing the Heun with step size control for arbitrarily dimensioned ODE systems. Maybe you can even simulate the n-body problem for large n? Or extend your simulation to three dimensions?